The Beauty of Numbers Project Pack

Made by the Learning team at RIBA

RIBA
Architecture.com

About this pack

- This project has been made to meet curriculum criteria for key stage 3 students (11 to 14-year olds) in Maths and Design Technology. As a general indicator, if students understand the keywords on the next slide they should be comfortable with the level of maths required.

- In this pack, you will find:
  - Some key information for adults to help understand how this project can be used to supplement and cover aims of school learning.
  - Slides for young people to read through and respond to in order to explore the impact mathematical sequencing and ratio have on architecture

- The project is designed to introduce students to the Golden Ratio and how this ratio may affect the design of buildings. Students will be expected to do tasks in a step-by-step approach to understand the mathematical principles being investigated, with answers on the next slide – please ask them not to go onto the next slide until they have attempted the task on the current slide!

Key information and aims

Keywords – make sure you know what these words mean	Equation Ratio Irrational Number Term (algebra definition) Architect Proportion
Materials – what you will need to collect to do this project.	Square Paper Calculator Plain Paper Ruler Pencil Colouring Pencils
Skills and knowledge you should have acquired by the end of this project. Can you explain or show things to other people?	I can explain what the Fibonacci Sequence is and how to calculate the numbers I can state the value of the Golden Ratio I can demonstrate the relationship between the Fibonacci Sequence and Golden Ratio in algebraic formula I can explain how architects can use the Golden Ratio in their designs I can design a building to a set of criteria

What the National Curriculum says children should learn:

Maths

- Use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- Substitute values in expressions, rearrange and simplify expressions, and solve equations
- Move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]
- Extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- Extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- Identify variables and express relations between variables algebraically and graphically

Design Technology

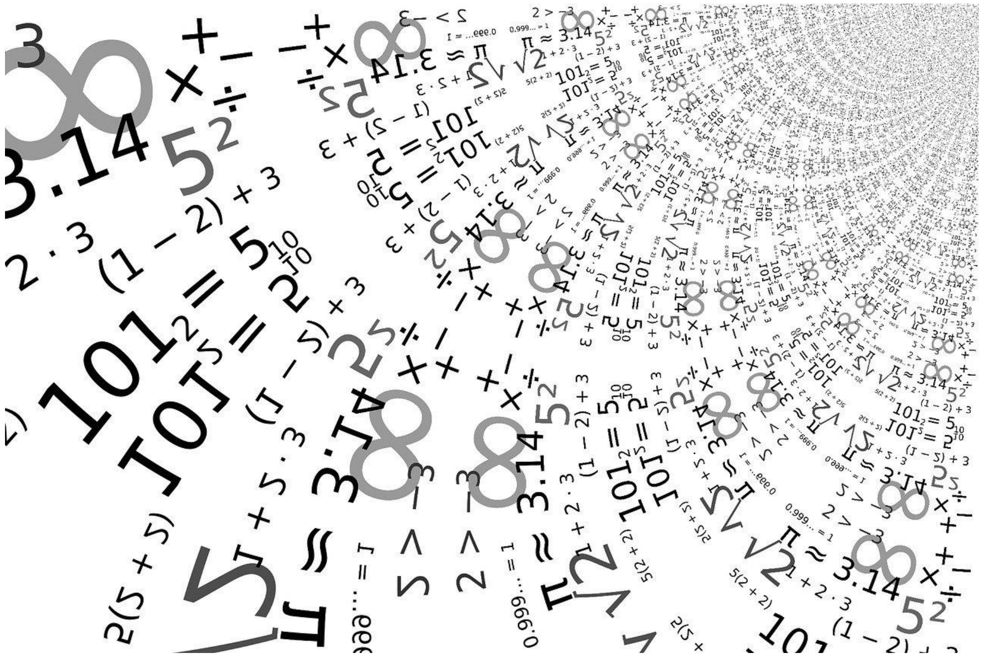
- Identify and solve their own design problems and understand how to reformulate problems given to them
- Develop and communicate design ideas using annotated sketches, detailed plans, 3-D and mathematical modelling, oral and digital presentations and computer-based tools
- Analyse the work of past and present professionals and others to develop and broaden their understanding

The Beauty of Numbers Design Challenge

In this project we are going to explore the hypothesis that some of the world's most beautiful buildings are based on mathematical principles.

We want you to understand the mathematical principles involved before using them to design your own beautiful building.

Please make sure you read each slide carefully and complete any tasks before going to the next one. Answers or explanations will be provided on the following slide for you to check your work, so please don't skip slides or go to the next one before finishing the current one.



The most beautiful buildings in the world?

Below are some buildings which are often said to be the most beautiful in the world. Do you recognise any? What do you think they have in common? Are there any shared features, shapes or styles? How old do you think they might be?

By weetoon66, Flickr.

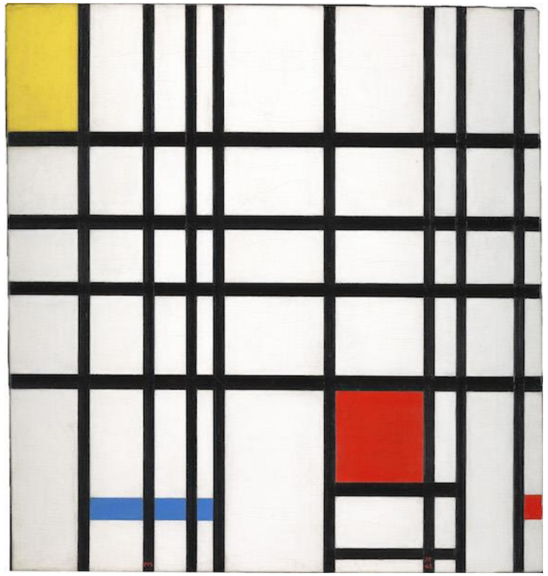
The Golden Ratio

Some architects believe that all the buildings you just saw were designed around the golden ratio.

The Golden Ratio is believed to be a ratio, which when applied to a building's dimensions, makes the design aesthetically pleasing to the eye. In maths it is often represented as the greek letter phi (ϕ).

The golden ratio has been used in many forms for centuries, ever since the Ancient Greek architect Phidias first discovered it and applied it to his work.

Now let's start to explore how this ratio is calculated...



The Sequencing of Numbers.

Have a look at the following sequence of numbers. Do you recognise it?

Task: Can you calculate what the next three numbers will be based on the numbers you have been provided?

1, 1, 2, 3, 5, 8, 13, ? ? ?

Hint: If you are stuck why not try looking at the numeric difference between each sequential number and see if there is a relationship between these numbers and the numbers in the sequence?

Were you right?

To work out the number you must calculate the sum of the two numbers before it.

For example to find the first missing number (21) you had to add 8 and 13 together.

1 1 2 3 5 8 13 $\boxed{21}$ $\boxed{34}$ $\boxed{55}$

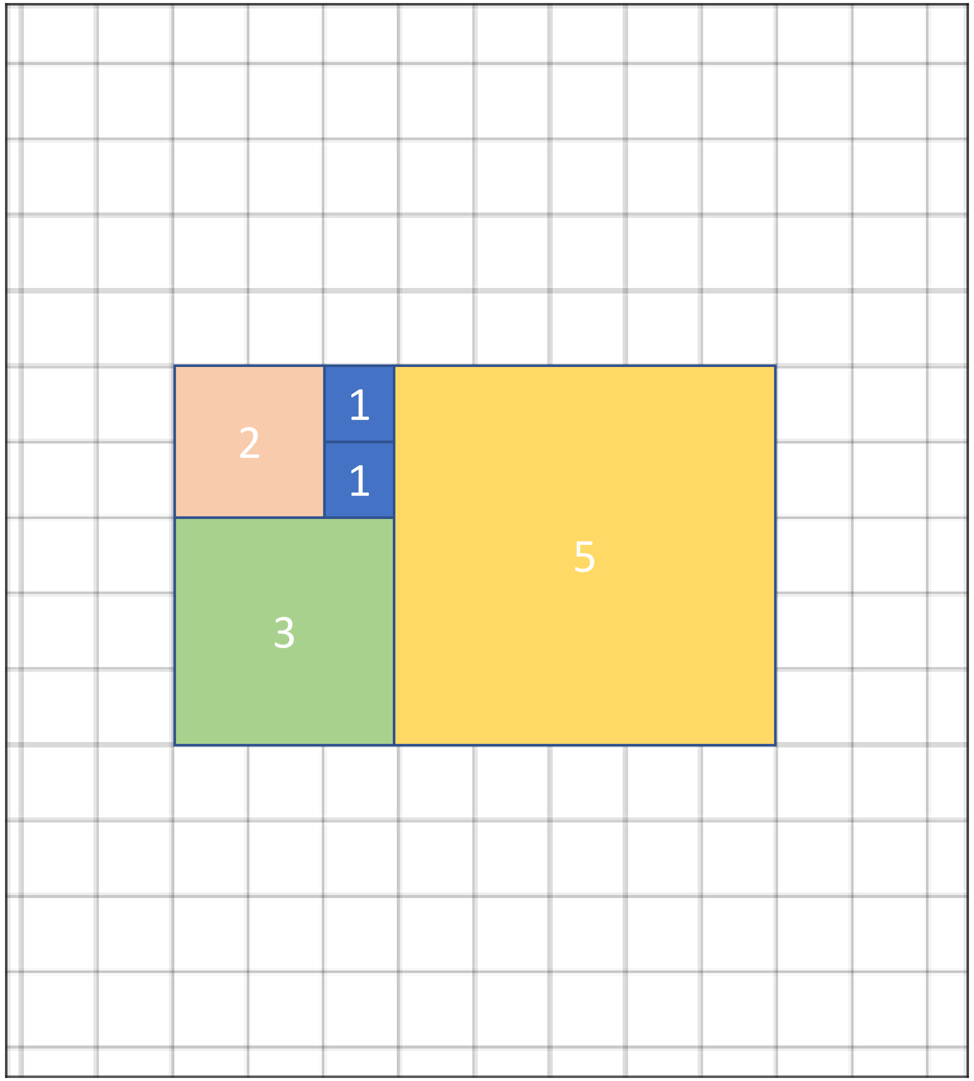
0 1 1 2 3 5 8 13 34

This is a famous sequence and has the name the **Fibonacci Sequence**

Let's make a visualisation of this sequence.

1. Grab some square paper
2. Draw and colour in one square.
3. Along one of the square's sides, draw and colour in another square.
4. Now look at the Fibonacci Sequence. Each time you draw and colour in the next square, the width and height will need to be the size of the next number in the sequence., e.g. 2x2 squares, 3x3 squares, 5x5 squares
5. When you draw each new square, one length of the square must run alongside the sides of the last two squares drawn.
6. Continue doing this until you run out of room on your paper.

Task: Think about how the squares' position might help you calculate the size of the next square. What is the connection?



Introducing algebra

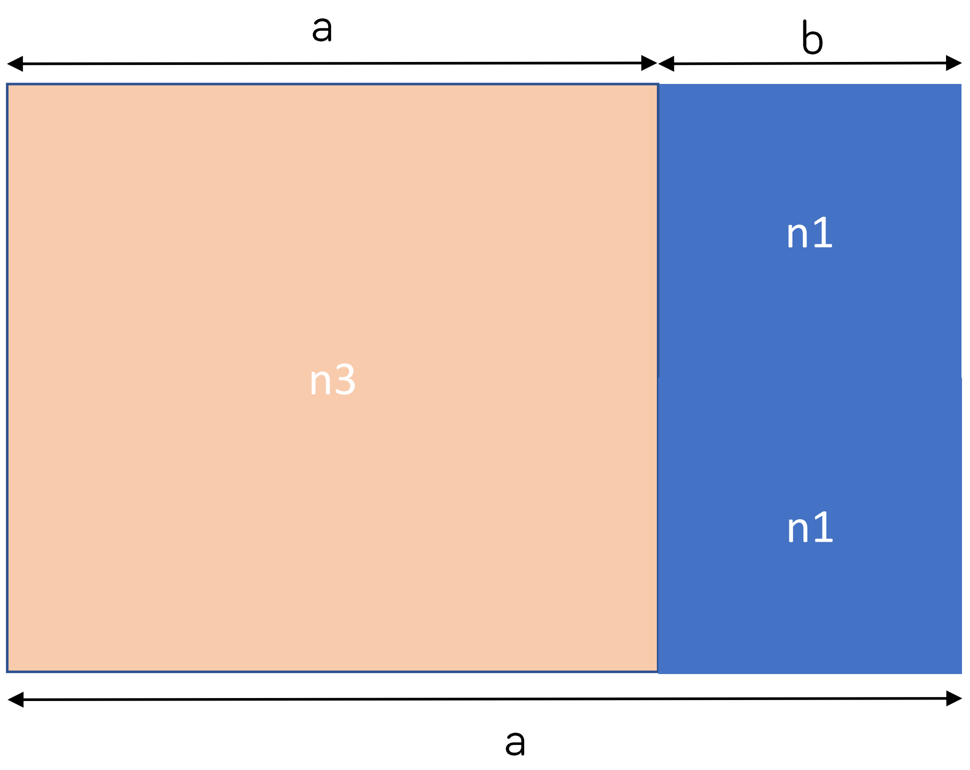
Every new rectangle you create by drawing a new square has its longest side equal the sum of the sides of the two previous squares drawn

a = a+b

We can also use algebra to express what the relationship is between the terms a and b.

$$\frac{a}{b} = \frac{(a+b)}{a}$$

Task: Can you expand the expression above?



Let's look a bit deeper...

The expanded expression is;

$$\frac{a}{b} = \frac{a}{a} + \frac{b}{a}$$

But let's focus on the left-hand fraction for a little bit $\frac{a}{b}$

If we start substituting the terms with values from the Fibonacci Sequence and then translating it into a numeric value, what do you think might happen?

A value	B value	Fraction	Value
1	1	1/1	1
2	1		
3	2		
5	3		
8	5		
13	8		

Task: Can you complete the table ? Please round the numeric number to show a maximum of 3 decimal places. You can use a calculator!

Noticing a pattern…

Have a look at the table and compare the answers to yours. How did you do?

In orange we have further continued the table for more of the Fibonacci Sequence.

What do you notice about the value calculated the further down the sequence you go?

A value	B value	Fraction	Value
1	1	1/1	1
2	1	2/1	2
3	2	3/2	1.5
5	3	5/3	1.667
8	5	8/5	1.6
13	8	13/8	1.625
21	**13**	**21/13**	**1.61**
34	**21**	**34/21**	**1.619**
55	**34**	**55/34**	**1.618**
89	**55**	**89/55**	**1.618**
144	**89**	**144/89**	**1.618**
233	**144**	**233/144**	**1.618**

The Magic Number!

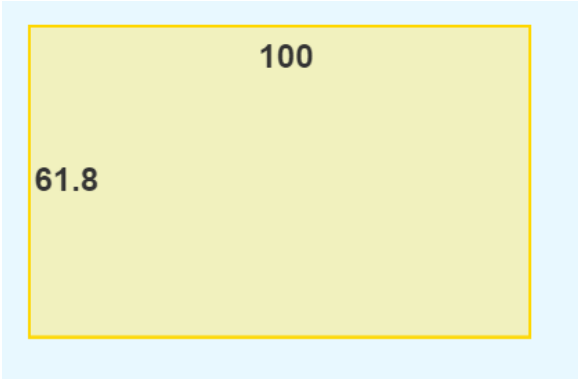
Although the values are rounded up to three decimal places (many of the values are irrational numbers), you will notice that the further down the sequence you go, the closer the number gets to 1.618.

Phi (ϕ) is also an irrational number, but we often refer to 1.168 as ϕ, just as we refer to π as 3.141

This number is very important as it is the number used in the Golden Ratio….

1: 1.618

So the rectangle to the right is a rectangle drawn in the Golden Ratio!



A perfectly proportioned rectangle according to the Golden Ratio!

Solve this!

Solve the following problem.

An architect wants to make a building 90m long and split it in two sections which use the Golden Ratio to determine the size of each section. At what distance along the 90m should they make this split? Show your workings out.

Hint: Look back at this equation $\frac{a}{b} = \frac{(a+b)}{a}$

We know $\frac{a}{b}$ = ϕ so we can rewrite this equation as $\phi = \frac{(a+b)}{a}$

We also know the total length is 90 (a+b) so you will need to rearrange the formula to work out a.

Applying the Golden Ratio

To calculate the place we would split the length we would need solve the equation by doing the following;

Write our equation out

$$\phi = \frac{(a+b)}{a}$$

Multiply both sides by a

$$a \times \phi = a + b$$

Divide both sides by ϕ

$$a = \frac{(a+b)}{\phi}$$

Solve the equation to get…

$$a = \frac{90}{\phi}$$

$$a = 53.5$$

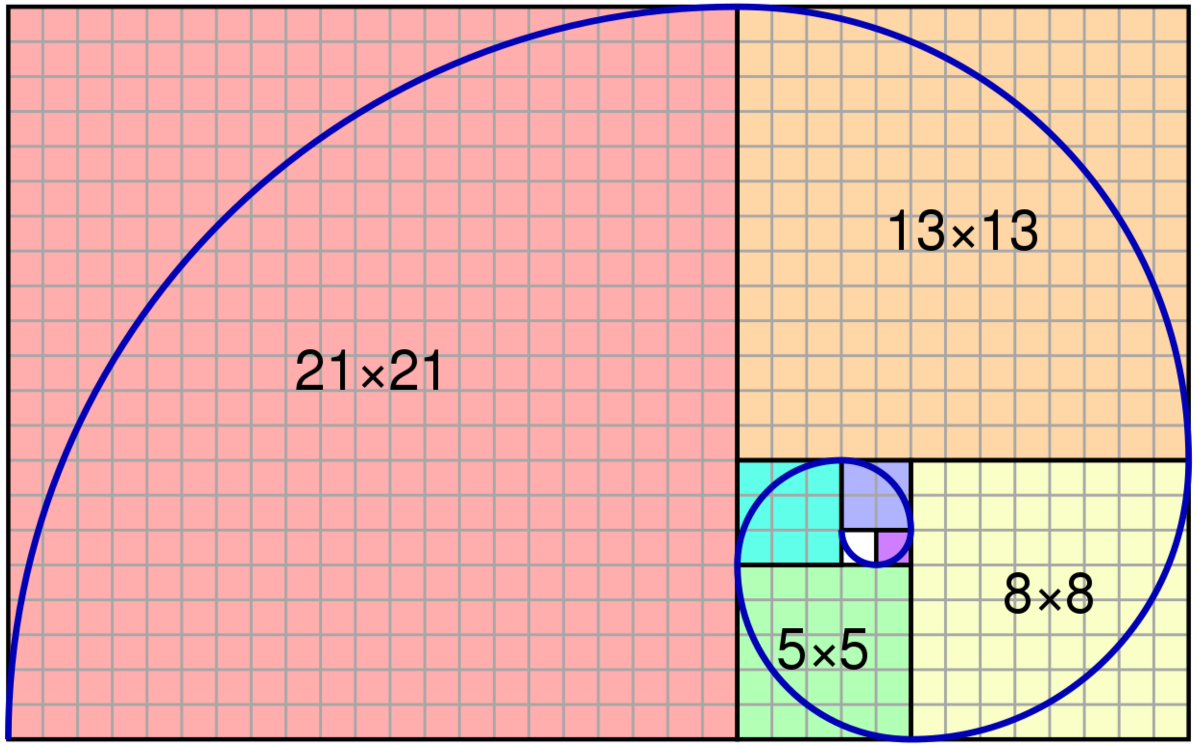
So the split would need to be 53.5m along the 90m length.
This would give two sections a = 53.5m and b = 36.5m

The Golden Spiral

The Golden Ratio is also linked to a special type of spiral which is meant to be very pleasing to the eye.

Go back to the squares you drew earlier. Starting in the corner of square one, draw a curve which goes from one corner to the opposite corner. Repeat in each square until you get a spiral shape that goes across all your squares.

Can you work out what the link between the spiral and the golden ratio is?



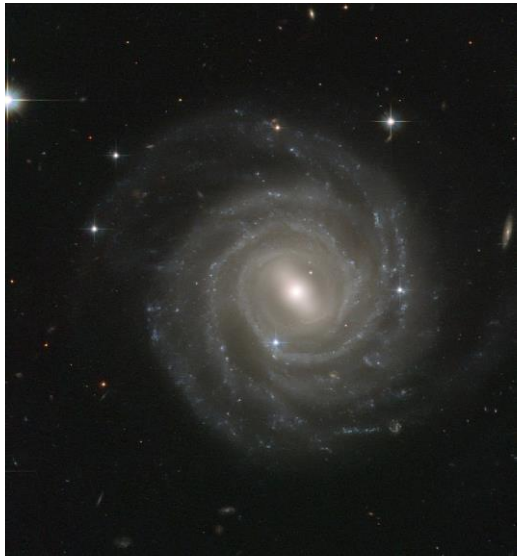
By Jahobr - Own work

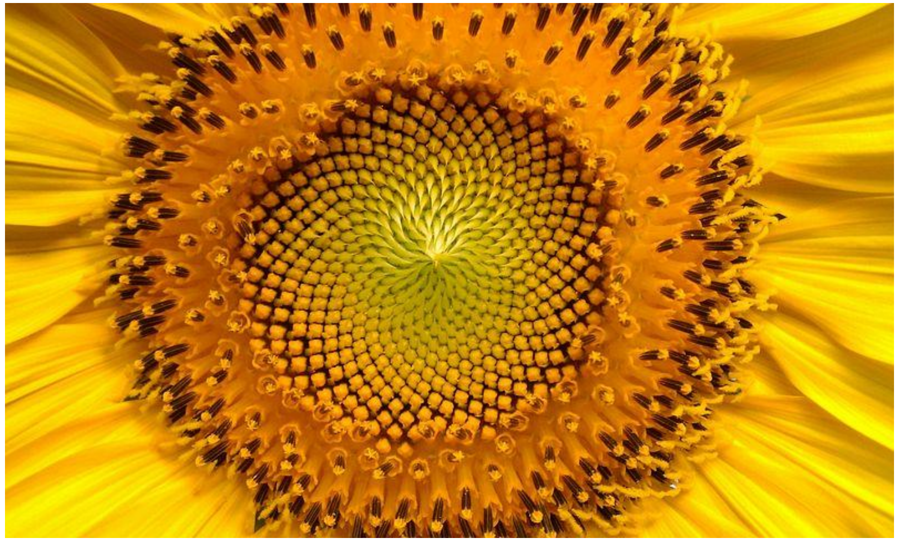
Spiralling Ways

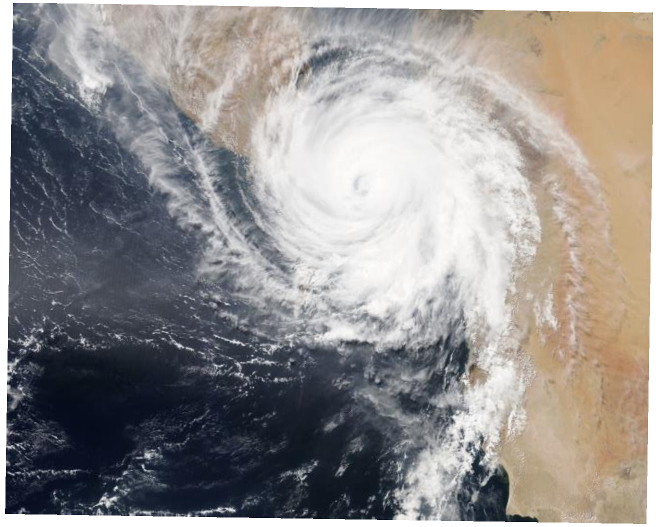
The Golden Spiral is linked to the Golden Ratio by the fact that the spiral's growth factor is φ.

Simply put, the spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes.

This special spiral is commonly found in nature and inspires lots of designs due to its supposed beauty!



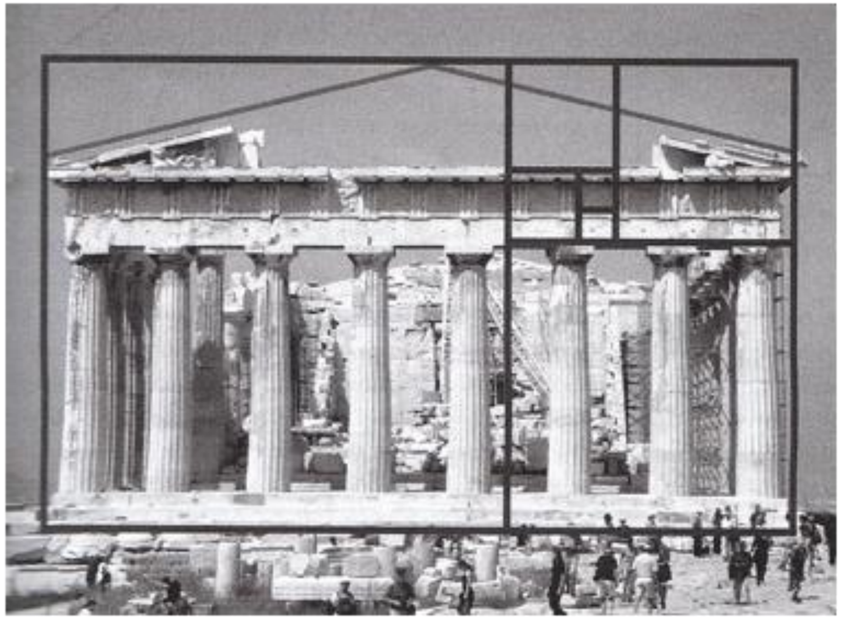


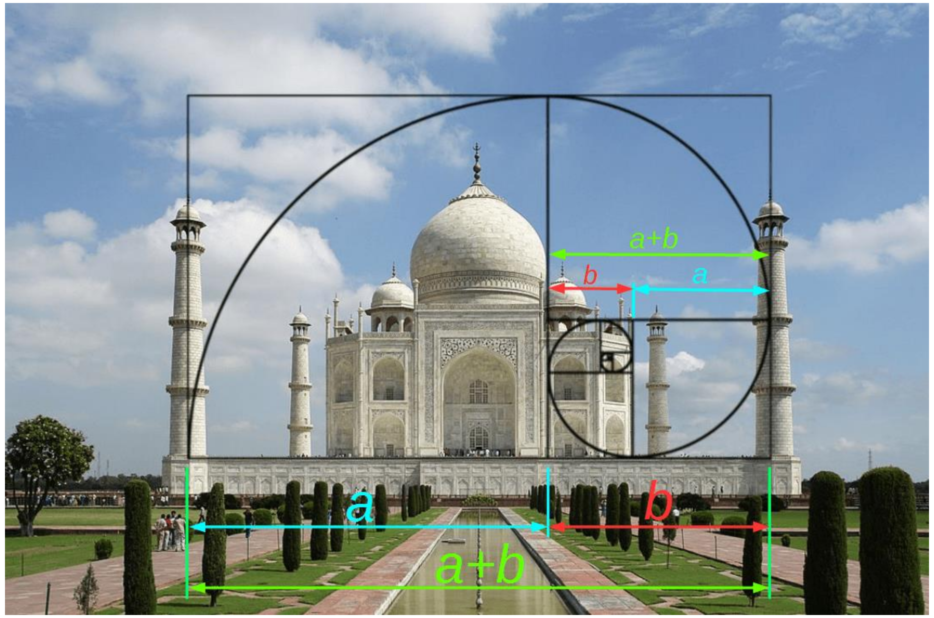


In Architecture

The images below show how some architects believe certain buildings have been designed based around the Golden Ratio. Their dimensions and features fit within the Golden Spiral, or within the Fibonacci Squares, so shows how different features are in proportion to each other.

What do you think? Do you agree the features are designed to the Golden Ratio? If so, do you think these have been deliberately designed in this way, or do you think it is a coincidence? Do you think they are beautiful for this reason, or do other things such as symmetry and style make them more beautiful?





By www.responisfy.com

Modern examples

Have a look at the following examples of architecture. Can you identify where they have used the Golden Ratio in their designs?

Hint: Look at the Fibonacci boxes and Golden Spiral from earlier to see shapes and proportions that might be used. Things that are positioned approximately two thirds of the length or height are likely to use the Golden Ratio.

By gb pandey from chandigarh, India - High Court,Chandigarh,India,

By Juerg.hug

Wallpaper Flare

By David Beasley, Flickr

Pixels

By Esther Westerveld, Flickr.

PXFuel

By Amanda SLater, Flickr

The Design Challenge!

Now that you have studied what the Golden Ratio is and how it can be applied to architecture, we want you to have a go at designing your own beautiful building which uses this Golden Ratio in some way.

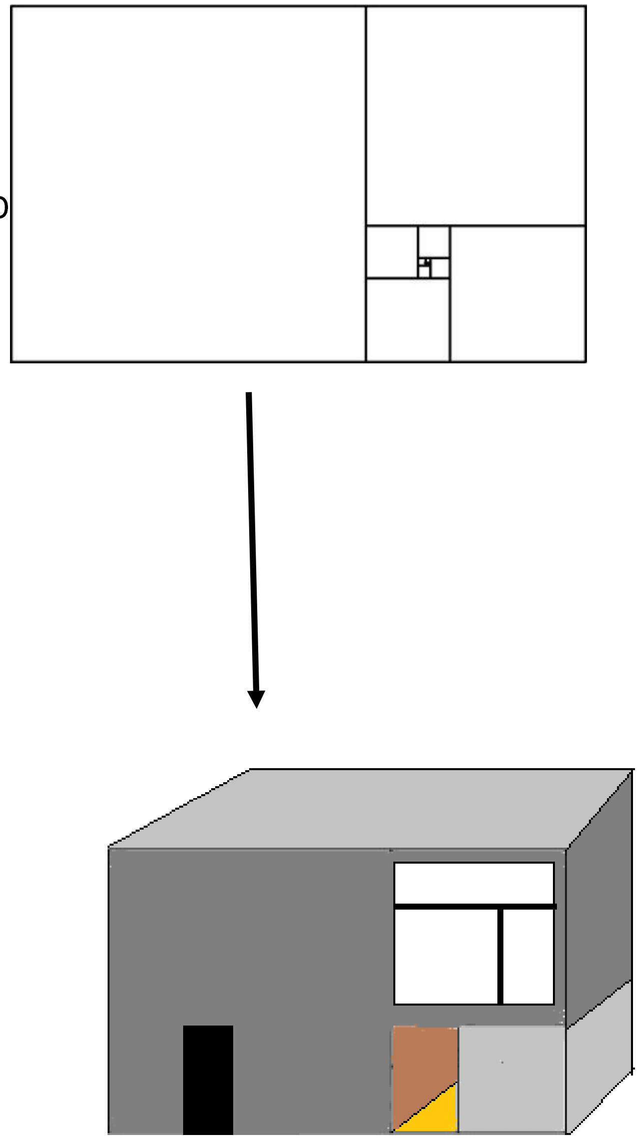
You could;

1. Split your building based on the Fibonacci Squares, adding features or having parts cut out/extended.
2. Use the spiral as a shape that can be repeated or used. Why not do a curved building or add spiral shaped features?
3. Look at applying the Golden Ratio to certain features of your building e.g. the roof, the windows, a balcony.

Things to think about;

1. Do you want your whole building to be based on the Golden Ratio or just parts? Maybe you want just a few features to be based on it?
2. Is it possible to turn your design into 3D?

Hint: Why not draw the Fibonacci boxes or spiral as gridlines you can then work to, as the picture to the right did?



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